On the Analysis and Design of Three Coupled Microstrip Lines

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Abstract **—The general configuration of a system of three coupled microstrip lines is analyzed and its parameters (mode numbers, effective dielectric constants, and modal impedances) are derived in terms of the system capacitances. These capacitances were computed by using the network analog method. In this paper, it is shown that for a system of three equal-width lines, there are, in general, five normal-mode impedances (four of which are independent). This would make the design of a system based on three equal-width lines, mainly with narrow strips and narrow separations, rather complicated. For a system of three equal-impedance lines, the number of mode impedances reduces to only three, which would facilitate the design. This latter system can be achieved by an appropriate increase of the width of the center line relative to that of the outer lines. Thus, a set of curves and a corresponding formula are obtained to determine this relative increase as a function of the geometric parameters. A complete set of design data for the new geometric configuration are also given. The experimental results of two fabricated three-line couplers, one based on the present data and the second on other published data, are compared.**

I. Introduction

The three coupled microstrip lines have many applications in communication systems and microwave components. Among these applications are coupler structures (e.g., [1]–[4]), the six-port reflectometer [5], and dc blocks [6].

In the analysis presented by Pavalidis and Hartnagel [1] for the three equal-width microstrip lines, and under the excitation they used, they reported that the characteristic impedance of the center line will always be the same as that of the side lines, and that there is no need to impose any condition for the capacitance coefficients of the network. Consequently, numerical results using the finite-difference methods were presented and only three modal impedances were obtained for the three equal-width coupled microstrip lines. Another analysis was introduced by Tripathi [7] in which the immittance parameters for the case of symmetrical three coupled microstrip lines or other inhomogeneous six-ports were derived in terms of the normal modes of the coupled system where only the analysis was dealt with.

In the present work, the authors found numerically that for three equal-width microstrip lines, the normal-mode impedances for the center line will be always greater than the normal-mode impedances of the outer lines. Hence, for this coupled system, there are five normal-mode impedances (four of which are independent), which would make the design of components based on three equal-width coupled lines (e.g., tightly coupled lines for which the width and separation are narrow) rather complicated. On the other hand, the design will be much easier for lines having equal impedances for each of the three normal modes of propagation, since only three modal impedances are encountered. To satisfy this requirement, it is found that the width of the center line should be increased relative to that of the outer lines. Hereafter, this system will be referred to as "the equal-impedance" system.

In Section II of this paper, the quasi-static parameters of the three present normal modes, namely the mode numbers, effective dielectric constants, and modal impedances, are found in terms of the self and mutual capacitances of the system in the filled and empty structures. In Section III, the numerical results are presented and some of them are compared with the only suitable data [1] that were available to the authors. This section also includes design curves for the suggested equal-impedance system. Section IV presents the experimental results of two fabricated three-line couplers, one based on the data obtained from [1] and the other based on the design curves presented in this paper. Conclusions are given in Section V.

II. Analysis

A system of three coupled microstrip lines, as shown in Fig. 1, can be described in terms of three normal modes [7] under the assumption of TEM propagation. These modes are denoted as A, B, and C modes. The analysis begins from the conventional telegraphist's equations for the filled and empty structures. Starting from these equations, we can arrive at the following relation:

$$[C^a]^{-1}[C^d][V_i] = (v_o/v_i)^2[V_i] \tag{1}$$

where $[C^d]$ and $[C^a]$ are the capacitance matrices of the system with and without dielectric, respectively.

The eigenvalues of (1) represent the effective dielectric constants of the system of Fig. 1 for the three propagating modes, and they will be denoted by ϵ_{ei}, where the subscript i represents the considered mode. The eigenvectors of (1) represent the mode numbers of the system of Fig. 1 for the three propagating modes and they will be denoted by $[R_i]$, where the subscript i represents the considered mode. The eigenvector of mode i will take the following form

$$[R_i] = \begin{bmatrix} 1 \\ R_{2i} \\ R_{3i} \end{bmatrix}. \tag{2}$$

Expressions for R_{2i} and R_{3i} were derived on the basis of (1) and they are given by

$$R_{2i} = [C_{13}C_{21} + C_{23}(\epsilon_{ei} - C_{11})] / [C_{12}C_{23} + C_{13}(\epsilon_{ei} - C_{22})] \tag{3}$$

$$R_{3i} = [\epsilon_{ei} - C_{11} - R_{2i}C_{12}]/C_{13} \tag{4}$$

where $C_{11}, C_{12}, \cdots$ are the elements of the matrix $[C]$, which corresponds to

$$[C] = [C^a]^{-1}[C^d].$$

Expressions for the modal impedances of this structure were derived and they are of the following form:

$$Z_i^{(1)} = 1/[v_i(C_{11}^d + R_{2i}C_{12}^d + R_{3i}C_{13}^d)] \tag{5}$$

$$Z_i^{(2)} = 1/[v_i(C_{22}^d + C_{12}^d/R_{2i} + R_{3i}C_{23}^d/R_{2i})] \tag{6}$$

$$Z_i^{(3)} = 1/[v_i(C_{33}^d + C_{13}^d/R_{3i} + R_{2i}C_{23}^d/R_{3i})] \tag{7}$$

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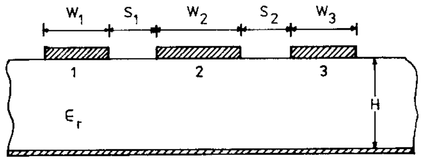


Fig. 1. Three coupled microstrip lines

where v_i is the mode velocity for mode i and the superscripts (1), (2), and (3) denote the line number as shown in Fig. 1.

Although the analysis is quite general, in the present work we are mainly interested in the cases where the two outer lines (1 and 3) are identical. For these cases, the eigenvectors for the present modes (mode A, mode B, and mode C, respectively) are

$$[R_A]=\begin{bmatrix}1\\R_{2A}\\1\end{bmatrix}\quad [R_B]=\begin{bmatrix}1\\R_{2B}\\1\end{bmatrix}\quad [R_C]=\begin{bmatrix}1\\0\\-1\end{bmatrix}. \tag{8}$$

It is clear from (6) and (8) that the impedance of the center line (2) for mode C is undefined. Thus, we will have, in general, five modal impedances for this present case. But since $Z_A^{(2)}/Z_A^{(1)} = Z_B^{(2)}/Z_B^{(1)}$ (as outlined by Tripathi [7]), only four of these impedances are independent.

As seen from the previous equations, the parameters of the three-line coupler can be obtained in terms of $[C^d]$ and $[C^a]$ only. These capacitances were computed by using the network analog method [8], which has also been used by other authors [2] for computations of a similar nature. The accuracy of this method is of the order of the discrete five-point Laplacian [8].

III. Numerical Results

A Fortran IV program has been written according to the analysis of Section II, for the general case of three asymmetric coupled lines. This program gives the effective dielectric constants, mode numbers, and the modal impedances. The program has been used to obtain the air and dielectric capacitance matrices for the three equal-width coupled lines of [9] and a good agreement of results was found. Also, the program was checked by removing one of the outer lines and the results were found to be the same as that obtained earlier by the authors for two asymmetric coupled lines [10].

For the case of three equal-width lines on an alumina substrate ($\epsilon_r = 9.8$, $H = 0.025$ in), the corresponding five modal impedances have been computed for different widths and separations and some of the results are plotted in Figs. 2–4. It is clear from Figs. 2 and 3, that the modal impedances of the center line are greater than the modal impedances of the outer lines, especially for mode A. This difference decreases as the width of the lines and/or their separation increases.

To have equal impedances for the three lines for each of the normal modes, the width W_2 of the center line should be increased. The value of W_2 necessary to satisfy this requirement was calculated by increasing W_2 in steps until equal modal impedances for the three lines were obtained. This was carried out for different values of $W_{1,3}/H$ and S/H. Then a set of curves was obtained for the ratio of W_2 to $W_{1,3}$ against S/H with $W_{1,3}$/H as a parameter (for an alumina substrate, $\epsilon_r = 9.8$; see Fig. 5). Curve fitting has been carried out for this set of curves, and a formula was obtained as a function of $W_{1,3}/H$ and S/H.

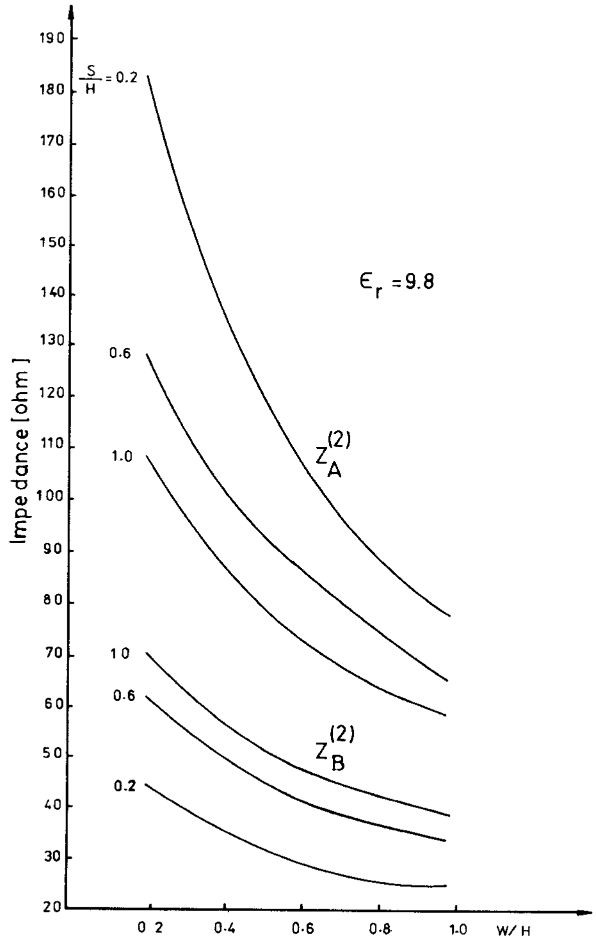


Fig. 2. Mode A and mode B impedances of the center line for the case of three equal-width lines

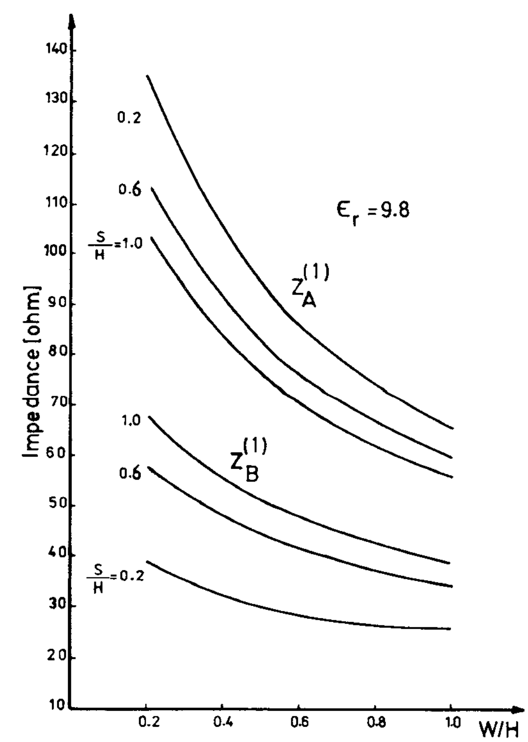


Fig. 3. Mode A and mode B impedances of the outer lines for the case of three equal-width lines.

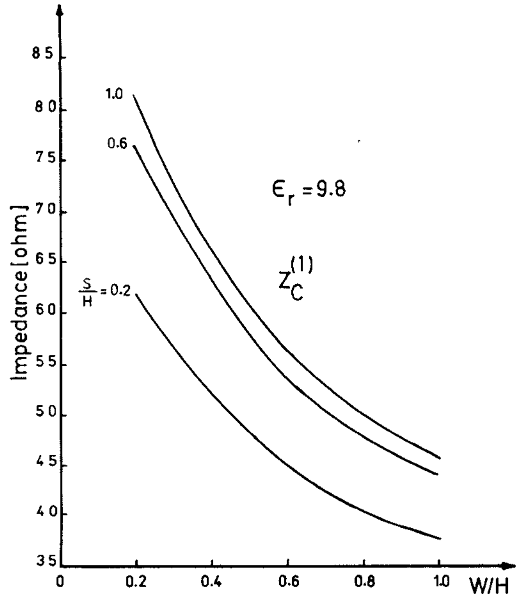


Fig. 4. Mode C impedance of the outer lines for the case of three equal-width lines.

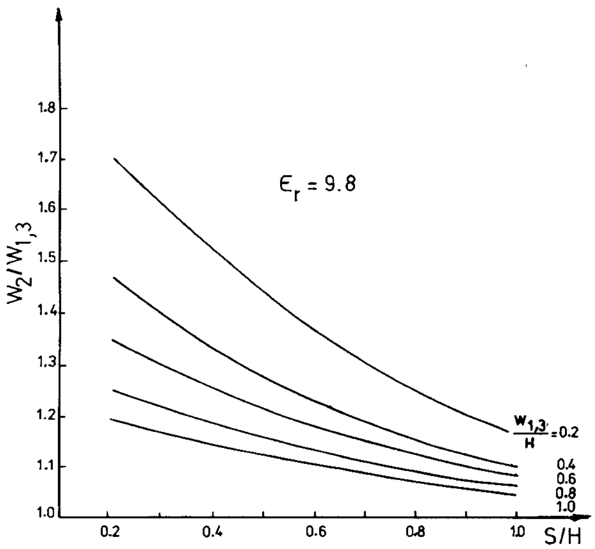


Fig. 5. The ratio of the widths of the central and outer lines for equal-mode impedances.

This formula is as follows:

$$W_2/W_{1,3} = \sum_{n=0}^{3} B_n (S/H)^n \tag{9}$$

where

$$B_0 = 2.40599 - 3.066312(W_{1,3}/H) + 3.463890(W_{1,3}/H)^2 - 1.668329(W_{1,3}/H)^3$$

$$B_1 = -1.412415 + 0.754368(W_{1,3}/H) + 1.984121(W_{1,3}/H)^2 - 1.498783(W_{1,3}/H)^3$$

$$B_2 = 0.610332 - 0.389012(W_{1,3}/H) - 0.178530(W_{1,3}/H)^2 - 0.424744(W_{1,3}/H)^3$$

$$B_3 = -0.206008 + 1.044009(W_{1,3}/H) - 2.235098(W_{1,3}/H)^2 + 1.74802(W_{1,3}/H)^3.$$

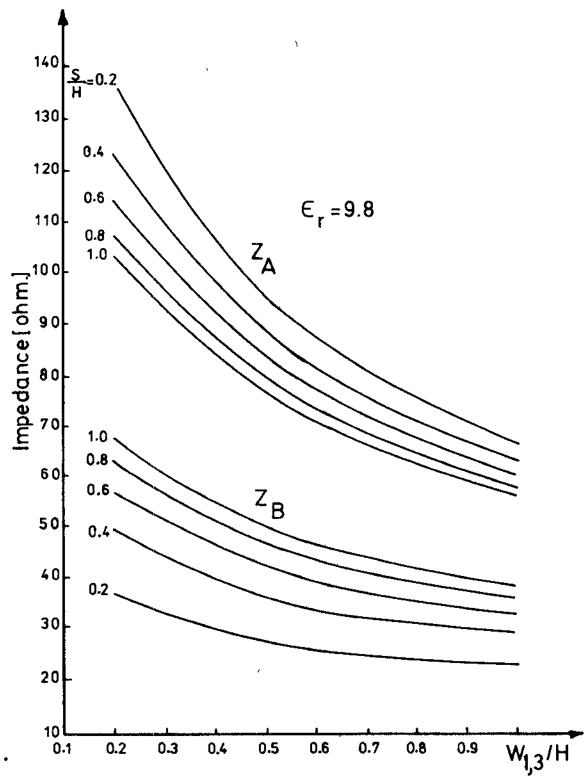


Fig. 6. Mode A and mode B impedances for the case of equal-mode impedance lines.

This formula was checked and found to be valid within the range $0.2 \leqslant [S/H, W_{1,3}/H] \leqslant 0.8$ with an error not greater than one percent when compared with the values used to plot the curves of Fig. 5. For values of S/H and $W_{1,3}/H > 0.8$, the accuracy of the above formula starts to decrease and reaches a maximum value of about 10 percent at $W_{1,3}/H = S/H = 1.0$. The curves of Fig. 5 should then be used for more accurate results. For such values ($W_{1,3}/H, S/H > 0.8$), it can also be deduced from Figs. 2–4 than equal-width lines start to have nearly equal modal impedances. In general, it was found that the error in determining the modal impedances is negligible when compared with the error in calculating the corresponding values of $W_2/W_{1,3}$.

The modal impedances for the three equal-impedances lines were calculated by making use of (9) (and Fig. 5 for $W_{1,3}/H, S/H > 0.8$) for different geometric parameters. The results are shown in Figs. 6 and 7. A sample of these results are compared with the results of [1], which were the most suitable available results for such a comparison, and this comparison is shown in Figs. 8 and 9. For complete information about the design data, the mode numbers and the effective dielectric constants for the normal modes of propagation are illustrated in Figs. 10 and 11 for both equal-impedance lines (solid line) and equal-width lines (dotted line). It should be noted that, in Figs. 10 and 11, the width W_2 of the center line for the equal-impedance lines can be determined either from Fig. 5 or (9).

The aforementioned results are for the quasi-static case. The effect of dispersion can be taken into consideration, only approximately, by using the dispersion model introduced by one of the authors [3]. In formulating this model, it was assumed that the mode number R_{2A}(m_1 in [3]) is equal to unity and that the mode number R_{2B}(m_2 in [3]) is equal to -2. It can be easily seen from the curves of Fig. 10(a) for R_{2A} that the above assumption is good for both equal-width and equal-impedance lines, although the latter have slightly higher values for R_{2A}. On

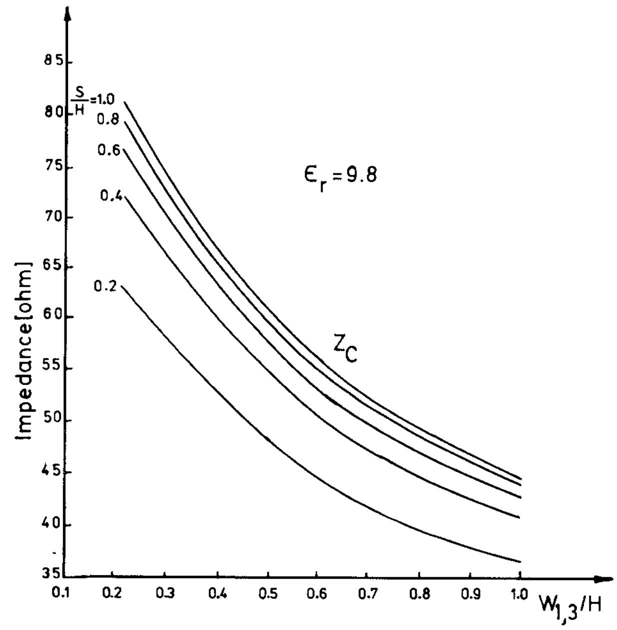


Fig. 7. Mode C impedance for the case of equal-mode impedance lines.

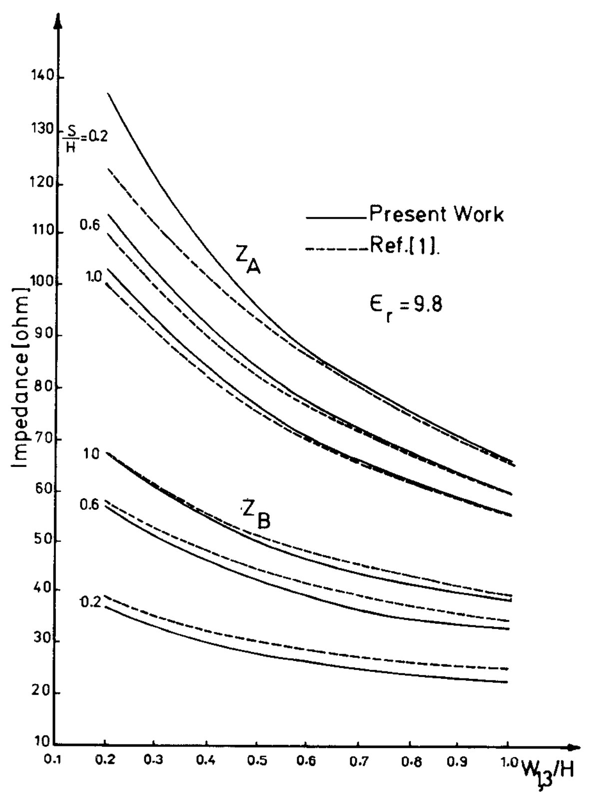


Fig. 8. Comparison of Z_A and Z_B for the case of equal-mode impedance lines with the results of [1]

the other hand, it is obvious from the curves of Fig. 10(b) for R_{2B} that the equal-impedance lines approach the above assumption more closely than the equal-width lines, especially for narrow strips and separations. Therefore, the dispersion model is expected to give, in general, better results for the equal-impedance lines.

It should be noted that the computations in this paper were carried out for the range $0.2 \leqslant S/H, W_{1,3}/H \leqslant 1.0$ to allow com-

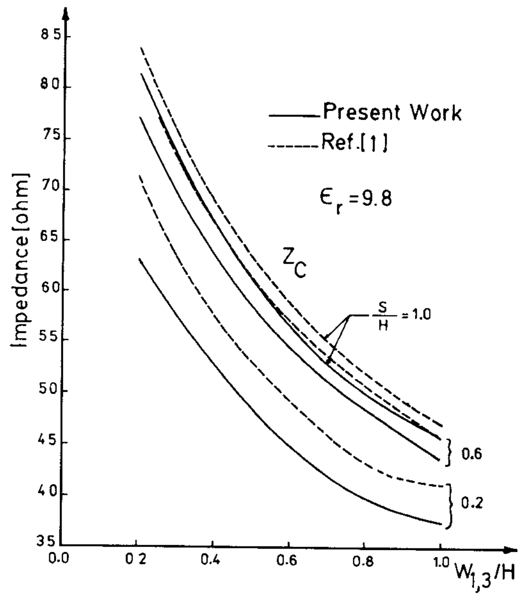


Fig. 9. Comparison of Z_C for the case of equal-mode impedance lines with the results of [1].

parison with the available published results [1]. However, the present method is quite general and can be used outside the aforementioned range.

IV. Experimental Results

The main aim of this section is to show by how much the width of the center line of a three-line coupler should be increased to have equal impedances for the three lines. Then, to compare the practical performances of both the original equal-width coupler and the resulting equal-impedance coupler. Therefore, two three-line couplers were manufactured. The dimensions of the first coupler were taken exactly the same as those reported for the 10-dB coupler of [1]. For the second coupler, the width of the outer lines W_1 and the separations S were kept the same as for the first coupler. Then the width W_2 of the center line necessary to equalize the impedances of the three lines was determined by using (9). The results are shown in the table of Fig. 12. The corresponding modal impedances for the equal-impedance case ($Z_A = 79\ \Omega$, $Z_B = 29.5\ \Omega$, and $Z_C = 48\ \Omega$) were found to be very near to those reported for the equal-width coupler [1] [$Z_A(Z_{ee}) = 78.4\ \Omega$, $Z_B(Z_{00}) = 30.0\ \Omega$, and $Z_C(Z_{0e}) = 50\ \Omega$]. This fact justified our trial to compare the practical performances of the two couplers. The configuration and the corresponding dimensions for the two couplers are given in Fig. 12. The coupler length L was calculated using the same approximate relation stated in [1]. The details of the configuration of the lines leading from the ends of the coupled lines to the outer measuring ports are not given in [1]. Therefore, the configuration used by Napoli and Hughes [11], shown in Fig. 12, was used for both couplers. This allows one to compare the performance of the two couplers under the same fabrication and measuring conditions. Thus, the following practical results, shown in Fig. 13, are good for the sake of comparing the present couplers and should not be directly compared with the reported practical results of [1].

The coupling coefficient between the central line and one of the outer lines (S_{12}), the isolation between the two outer lines (S_{13}), and the return loss for one of the outer lines (S_{11}) were measured in the frequency range from 2.5 to 5.5 GHz. The results for both couplers are compared in Fig. 13. It is clear from this

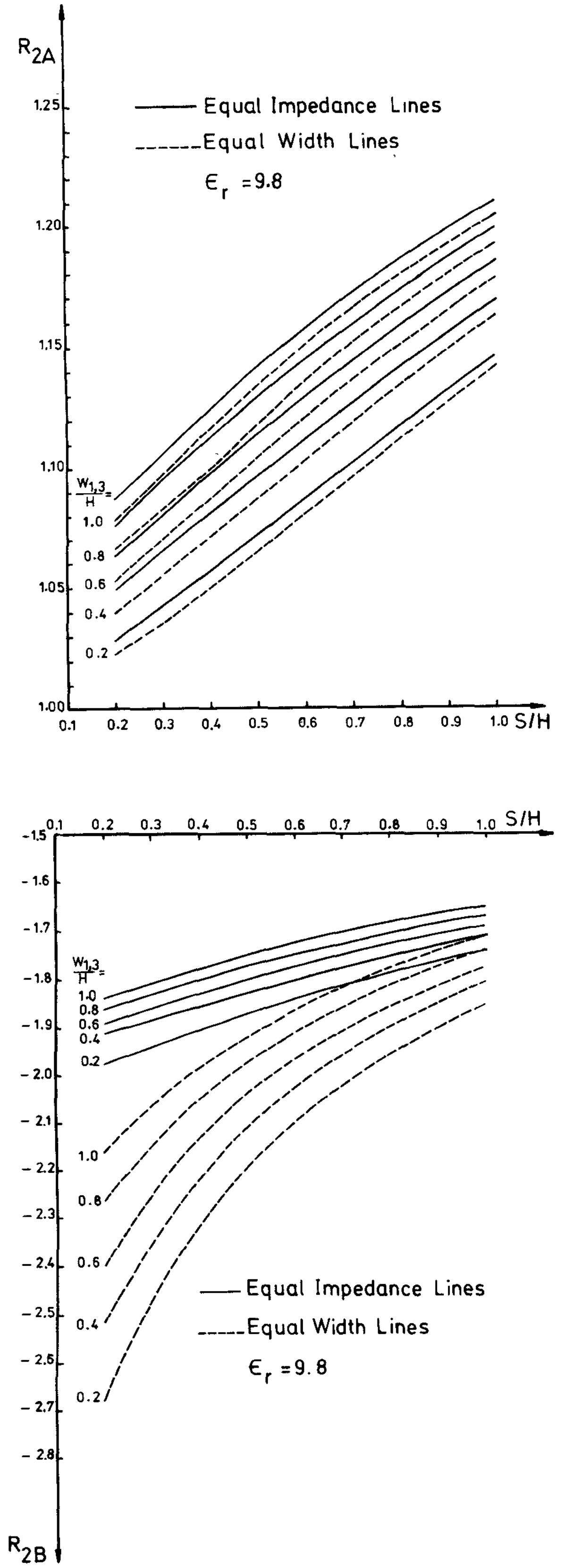


Fig. 10. (a) Variation of the mode number R_{2A} with the geometric dimensions. (b) Variation of the mode number R_{2B} with the geometric dimensions.

figure that the variation of the coupling coefficient with frequency is smaller for the coupler based on equal-impedance lines when compared with that based on the data of [1], especially at the higher frequencies. Also, it can be noticed that the isolation for

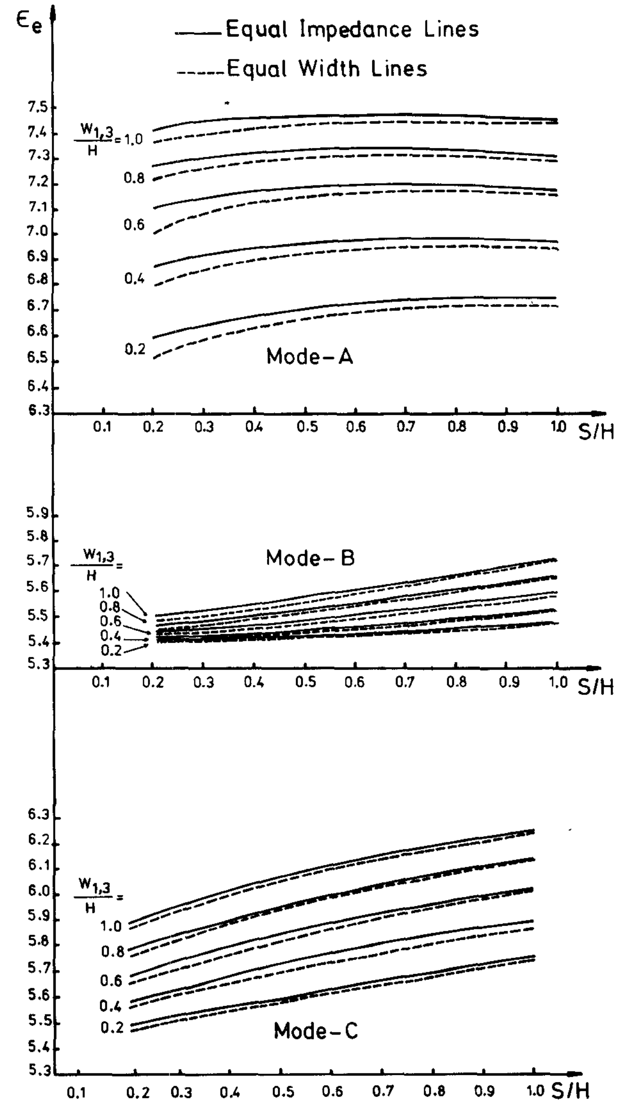
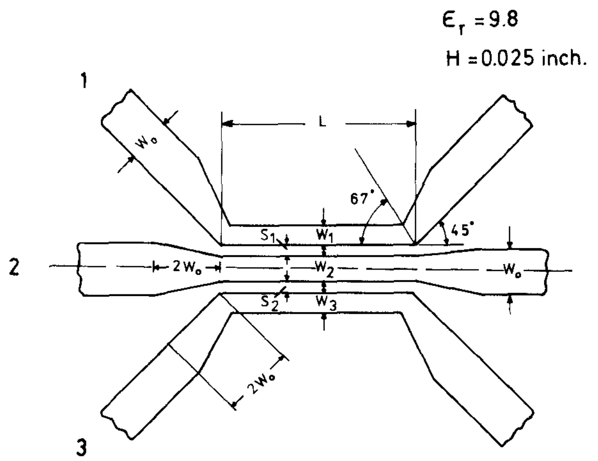


Fig. 11 Variation of the effective dielectric constants with the geometric dimensions.

the coupler based on the equal-impedance lines is better within nearly the whole frequency range. The return loss from one of the side ports was found to be almost the same for both couplers.

V. Conclusions

A general analysis for three coupled microstrip lines was given. It was shown that a system of three equal-width lines has five normal-mode impedances (four of which are independent). For such a system, the corresponding modal impedances were calculated for different geometric dimensions. To have equal modal impedances for the three lines, it is necessary to increase the width of the central line. A set of curves and a corresponding formula were given which allow one to determine this necessary width. Then a set of design curves for the equal modal impedance system was given. The difference in modal impedances between the equal-width lines and the equal-impedance lines was found to be large, particularly for mode A and at small widths and separations. On the other hand, the parameters of the equal-impedance and equal-width systems approach each other as the width and separation increase. In the limit (W/H and $S/H > 0.8$), the two systems will be practically indistinguishable from each other. The comparison of the experimental results of two 10-dB couplers, one based on the available published data and the other



Geometric Dimensions	$W_{1,3}$ (μm)	W_2 (μm)	$S_{1,2}$ (μm)	L (mm)	W_o (μm)
Coupler of Ref [1]	431 8	431 8	190.5	7.690	600
Present Coupler	431.8	550.23	190.5	7.589	600

Fig 12. Configuration and dimensions of the two couplers

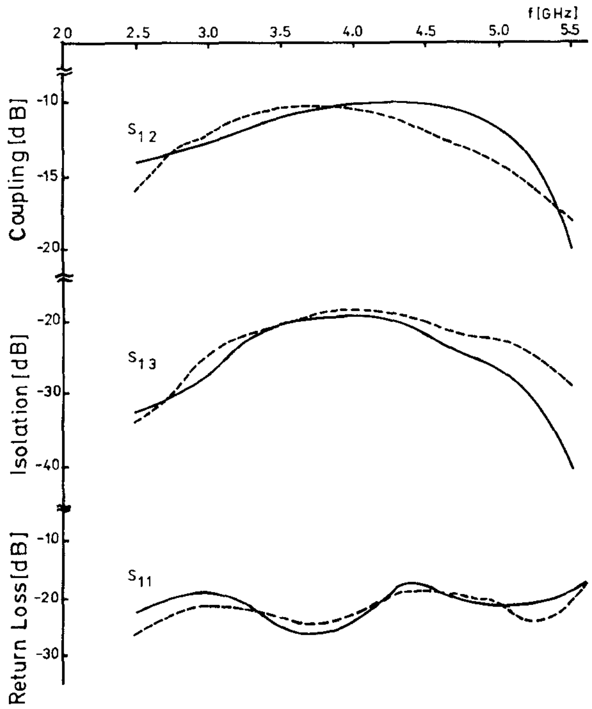


Fig. 13. Comparison of the behavior of the three-line coupler of this paper —— and that of [1]---.

on the present equal-impedance lines, showed that the latter gives, in general, a better behavior for the present example. In the present work, no attempt has been made to achieve a superior performance of the equal-impedance system over the equal-width system or vice versa.

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References

[1] D. Pavalidis and H. L. Hartnagel, "The design and performance of three-line microstrip couplers," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-24, pp. 631–640, Oct. 1976.
[2] V. Tulaja, B. Schiek, and J. Köhler, "An integrated 3-dB coupler with three strips," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-26, pp. 643–645, Sept. 1978.
[3] R. J. Collier and N A. El-Deeb, "Microstrip coupler suitable for use as a 6-port reflectometer," *Proc. Inst. Elec. Eng.*, vol. 127, pt. H, pp. 87–91, Apr 1980.
[4] L. Gruner, "Nonsymmetrical three-line microstrip couplers," in *Proc. Eur Microwave Conf.*, A 10-3, 1981, pp. 844–849.
[5] R. J. Collier and N. A. El-Deeb, "On the use of a microstrip three-line system as a six-port reflectometer," *IEEE Trans. Microwave Theory Tech.*, vol MTT-27, pp. 847–853, Oct. 1979.
[6] V. K. Tripathi, Y K Chin, and H Lee, "Interdigital multiple coupled microstrip dc blocks," in *Proc. Eur. Microwave Conf.*, A8-4, 1982, pp. 632–636.
[7] V. K. Tripathi, "On the analysis of symmetrical three-line microstrip circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-25, pp. 726–729, Sept. 1977.
[8] B. L. Lennartsson, "A network analogue method for computing the TEM characteristics of planar transmission lines," *IEEE Trans. Microwave Theory Tech*, vol. MTT-20, pp. 586–591, Sept. 1972.
[9] F.- Y. Chang, "Transient analysis of lossless coupled transmission lines in a nonhomogeneous dielectric medium," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-18, pp. 616–626, Sept. 1970.
[10] N. A El-Deeb, E. A. F. Abdallah, and M. B. Saleh, "Design parameters of inhomogeneous asymmetrical coupled transmission lines," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 592–596, July 1983.
[11] L. S Napoli and J J. Hughes, "Characteristics of coupled microstrip lines," *RCA Rev.*, pp. 479–498, Sept 1970.

Broad-Band Permittivity Measurements Using the Semi-Automatic Network Analyzer

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***Abstract*—This paper outlines the use of the network analyzer to measure the dielectric properties of materials over a broad frequency range. The method described here is based on transmission techniques with simple procedures for obtaining initial estimates and unambiguous solutions for the dielectric parameters. A further feature is that this measurement technique provides a degree of self-checking for inconsistent results.**

I. Introduction

The semiautomatic network analyzer (SANA) is a powerful tool for the measurement of the permittivity of materials. Most of the classical techniques [1] for permittivity measurements can be adapted for the SANA, and with the latest generation of equipment, very broad-band measurements can be done very efficiently. The technique described here is based on transmission measurements, and can be applied to fully or partially filled guide, although the latter method is more complex.

II. Measurement Theory

With suitable calibration standards and procedures, the SANA can be used to measure samples mounted in coaxial line, waveguide, or stripline-type structures. In general, the coaxial and waveguide methods will provide the best accuracy since precision calibration standards are available for these systems. The coaxial line provides the maximum measurement bandwidth, but sample mounting is often difficult with this technique.

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